ROBUST NONLINEAR CONTROL OF A CLASS OF NONLINEAR PROCESSES: APPLICATION TO WASTEWATER TREATMENT

V. GONZÁLEZ-ÁLVAREZ†, J.P. GARCÍA-SANDOVAL†,
C. NAVARRO-FITCH† and B. CASTILLO-TOLEDO‡

†*Chemical Engng. Department, Universidad de Guadalajara, Guadalajara, Jalisco 44430, México.*
[paulo.garcia],[victor.ga]@cucei.udg.mx
‡*CINVESTAV-IPN Unidad Guadalajara, Zapopan, Jalisco 44019, México.*
toledo@gdl.cinvestav.mx

Abstract— **In this paper we propose a robust error feedback controller for nonlinear bioprocesses that allows us to track predetermined constant and/or oscillatory profiles while attenuating the disturbances and maintaining the stability conditions of such bioprocesses. Various numerical case studies for an anaerobic digestor model are conducted to test the robustness properties of the proposed controller. It is found that the proposed controller yields excellent responses in the face of parameter uncertainties, load disturbances and set-point changes.**

Keywords— **Robust control, Nonlinear processes, Anaerobic digestion.**

I INTRODUCTION

The severity of the non-linearities in bioprocesses influences the selection of control algorithms for their successful regulation or trajectory tracking control. Control strategies based on a linearized model have shown to yield unsatisfactory performances if the process is subject to large disturbances or significant set-point changes. In addition, the wide range of operating conditions encountered in start-up, shut-down or trajectory tracking of bioprocesses, also pose an important challenge for the application of nonlinear control techniques. In the last two decades, a number of nonlinear control schemes, ranging from nonlinear control based on differential geometric approach (Kravaris and Kantor, 1990), nonlinear model predictive control (Patwardhan *et al.*, 1990) and generic model control (Lee and Sullivan, 1988), have been developed to overcome such problems with limited success since they largely rely on the availability of a good process model, which is not always easy to obtain. In the particular case of bioprocesses, these are complex with poorly understood bioreaction kinetics which usually lead to models with uncertain and/or time varying parameters. These cases are best handled with robust nonlinear control strategies in order to fully meet closed-loop objectives such as tracking, regulation and disturbance attenuation.

In this work, a robust nonlinear model-based control technique is proposed to track predetermined trajectories of nonlinear dynamic bioprocesses under the influence of uncertain parameters and load disturbances. This robust regulator, is an error feedback controller which relies on the existence of an internal model, obtained by finding, if possible, an immersion of the exosystem dynamics into an observable one, which allows to generate all the possible steady state inputs for the admissible values of the system parameters (Isidori, 1995). We illustrate the performance of the proposed control scheme by applying it, via numerical simulations, for the trajectory tracking and disturbance attenuation in an anaerobic digestion (AD) process under the most uncertain conditions. This paper is organized as follows: Section II gives an overview of the theory behind the robust control scheme; a nonlinear dynamical model for a typical AD process is presented in Section III; and the error feedback controllers are developed for three study cases: regulation control, oscillatory disturbance rejection and trajectory tracking. Simulation results and discussion are presented in Section V. Finally, we close the paper with some concluding remarks.

II ROBUST REGULATION PROBLEM FOR NONLINEAR SYSTEMS

Let us consider the following nonlinear system

$$\dot{x} = f(x,u,w,\lambda), \quad (1)$$
$$\dot{w} = s(w), \quad (2)$$
$$e = h(x,w,\lambda), \quad (3)$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ are the state and input variables of the process, respectively; $\lambda \in \mathcal{R}^s$ denotes a parameter vector which may take values in a neighborhood $\wp \subset \mathcal{R}^s$ of the nominal ones; $w \in \mathcal{R}^q$ represents the state of an external signal generator -the exosystem-, which models the reference and disturbance signals affecting the process. Finally, the last equation describes the tracking error $e \in \mathcal{R}^p$ which in many cases is given as the difference between the system output and the reference signal.

The *Error Feedback Regulation Problem* for the aforementioned system is defined as the problem of tracking the reference signals and/or rejecting the disturbance signals, while maintaining the closed-loop stability property under the influence of varying parameters in a neighborhood of the nominal values. This problem may be solved by determining a certain submanifold of the state space (x, w), where the tracking error is zero, which is rendered attractive and invariant by feedback. To be more precise, the *nonlinear robust regulation problem* (NRRP) consists in finding, if possible, a dynamic controller of the form

$$\begin{aligned} \dot{z} &= \varphi(z, e), \\ u &= \vartheta(z) \end{aligned}$$

such that, for all admissible values λ in a neighborhood $\wp$ of the nominal values, the following conditions hold

N1 Stability: The equilibrium point $(x, z) = (0, 0)$ of the closed-loop system without disturbances
$\dot{x} = f(x, \vartheta(z), 0, \lambda)$,
$\dot{z} = \varphi(z, h(x, 0, \lambda))$,
is asymptotically stable.

N2 Regulation: For each initial condition $(x(0), z(0), w(0))$ in a neighborhood of the origin, the solution of the closed-loop system
$\dot{x} = f(x, \vartheta(z), w, \lambda)$,
$\dot{z} = \varphi(z, h(x, w, \lambda))$,
$\dot{w} = s(w)$,
satisfies the condition $\lim_{t\to\infty} e(t) = 0$.

An instrumental assumption in the solution of the NRRP is:

Assumption 1 *The equilibrium point $w = 0$ is stable in the Lyapunov sense, and all the eigenvalues of $S = \frac{\partial s}{\partial w}\big|_{w=0}$ lie on the imaginary axis.*

Isidori and Byrnes (1980) gave first a solution to the problem NRRP, as stated in the following result:

Theorem 1 *(Isidori, 1995) Assume (1) holds. Then, the nonlinear Robust Regulation Problem is solvable if and only if there exist mappings*

$$x_{ss} = \pi(w, \lambda), \quad \text{and} \quad u_{ss} = \gamma(w, \lambda) = \begin{pmatrix} \gamma_1(w, \lambda) \\ \vdots \\ \gamma_m(w, \lambda) \end{pmatrix},$$

with $\pi(0, \lambda) = 0$ and $\gamma(0, \lambda) = 0$, both defined in a neighborhood of the origin, satisfying the equations

$$\frac{\partial \pi(w, \lambda)}{\partial w} s(w) = f(\pi(w, \lambda), \gamma(w, \lambda), w, \lambda) \quad (4)$$

$$0 = h(\pi(w, \lambda), w, \lambda) \quad (5)$$

for all (w, λ), and such that for each $i = 1, \ldots, m$ the exosystem is immersed into a system

$$\dot{\zeta} = \varphi(\zeta), \zeta \in \mathcal{R}^d \quad (6)$$

$$\gamma(w, \mu) = \psi(\zeta) \quad (7)$$

defined on a neighborhood Ξ^0 of the origin, in which $\varphi(0) = 0$ and $\psi(0) = 0$, and the two matrices

$$\Phi_0 = \left[\frac{\partial \varphi}{\partial \zeta}\right]_{\zeta=0,}, \qquad H_0 = \left[\frac{\partial \psi}{\partial \zeta}\right]_{\zeta=0,}, \quad (8)$$

are such that the pair

$$\begin{pmatrix} A_0 & 0 \\ NC_0 & \Phi_0 \end{pmatrix}, \qquad \begin{pmatrix} B_0 \\ 0 \end{pmatrix} \quad (9)$$

is stabilizable for some choice of the matrix N, and the pair

$$\begin{pmatrix} C_0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} A_0 & -B_0 H_0 \\ 0 & \Phi_0 \end{pmatrix} \quad (10)$$

is detectable. Here,

$$\begin{aligned} A_0 &= \left[\frac{\partial f(x, u, w, 0)}{\partial x}\right]_{x=0, w=0, u=0}, \\ B_0 &= \left[\frac{\partial f(x, u, w, 0)}{\partial u}\right]_{x=0, w=0, u=0}, \\ C_0 &= \left[\frac{\partial h(x, w, 0)}{\partial x}\right]_{x=0, w=0}. \end{aligned}$$

Thus, the controller that solves the problem is

$$\begin{aligned} \dot{\xi} &= A_0 \xi + B_0 [u(t) - \psi(\zeta)] - G_1 (C_0 \xi - e) \\ \dot{\zeta} &= \varphi(\zeta) - G_2 (C_0 \xi - e) \\ u(t) &= K\xi + \psi(\zeta) \end{aligned} \quad (11)$$

where K is chosen to render $(A_0 + B_0 K)$ as Hurwitz, while G_1 and G_2 must guarantee that

$$\begin{pmatrix} A_0 - G_1 C_0 & -B_0 H \\ -G_2 C_0 & \Phi_0 \end{pmatrix} \quad (12)$$

is also Hurwitz. $\odot$

Remark 1 *Equations (4) are known as the Francis-Isidori-Byrnes equations (FIB) (Delli Priscoli et al., 1997) which are used to find the subset Z on the Cartesian product $\mathcal{R}^n \times \mathcal{R}^q$, called the zero tracking error submanifold, where mapping $x_{ss} = \pi(w, \lambda)$ represents the steady state zero output submanifold and $u_{ss} = \gamma(w, \lambda)$ is the steady state input which makes x_{ss} invariant.*

In most I/O linearization control approaches u_{ss} is usually calculated by using Lie brackets; however, in the particular case of regulation, it is extremely difficult to find the steady state input since most typical state or error feedback controller designs (such as

I/O linearization) do not take into account unmodeled dynamics or time varying parameters. Instead, one may find a differential equation (free of uncertain parameters) that represents an immersion of the exosystem into an observable system, which can generate, for some appropriate initial conditions, the exact steady-state input for all the values of the parameter in a suitable neighborhood. Since these initial conditions are also unknown, the structure of the immersion is used in the controller to asymptotically estimate the required steady-state input allowing the controller to incorporate the desirable robustness properties. In this regard, Castillo Toledo *et al.* (2004) have shown that by allowing the immersion to depend explicitly on the exosystems states w, this calculation may be alleviated for an extended class of functions, including trigonometric ones, which constitute the so-called generalized immersion. Moreover, Huang (2001) has shown that if the steady state input is given by a polynomial in w, then it is always possible to find a linear immersion given by

$$\begin{aligned} \dot{\zeta} &= \Phi\zeta, \zeta \in \mathcal{R}^r, \\ \gamma(w,\mu) &= H\zeta. \end{aligned} \quad (13)$$

III THE PROCESS MODEL

The last two decades have witnessed an increasing interest in the application of advanced control techniques to the wastewater treatment field since the involved bioprocesses require careful monitoring in order to fulfill the requirements related to water quality and ecological norms (Dochain and Vanrolleghem, 2001). However, the optimal control of wastewater treatment processes, such as anaerobic digestion (AD), faces important uncertainties arising from the intrinsic complexity of the plant design.

AD is a multistep biological process in which complex organic matter is degraded into a gas mixture of CH_4 and CO_2. It reduces the inlet organic matter by using acidogenic bacteria and methanogenic archae to produce valuable energy (i.e., CH_4) (Henze *et al.*, 1995). When AD is performed in continuous biofilm-reactors, the acidogenic phase can be described by the following two ordinary differential equations (Bernard *et al.*, 2001):

$$\dot{X} = (\mu - \alpha D) X \quad (14)$$

$$\dot{S} = (S_{in} - S) D - \frac{\mu X}{Y} \quad (15)$$

where X, S and S_{in} are, respectively, the concentrations of acidogenic bacteria, chemical oxygen demand (COD), and inlet COD. α $(0 \leq \alpha \leq 1)$ denotes the biomass fraction that is retained by the bioreactor bed (i.e., $\alpha = 0$ for the ideal fixed-bed reactor and $\alpha = 1$ for the ideal continuous stirred tank reactor), Y is the biomass yield coefficient for COD degradation and $D = D(t) \geq 0$ denotes the dilution rate and it is assumed to be bounded, i.e., $D^- \leq D(t) \leq D^+$. The specific growth rate is given by the nonlinear Monod equation in which most parameters are badly or inadequately known (Dochain and Vanrolleghem, 2001; Vanrolleghem and Dochain, 1998):

$$\mu = \mu_{\max} S/(K_s + S) \quad (16)$$

where $\mu_{\max}$ and K_S are the maximum specific growth rate and the half saturation parameter associated with S, respectively. It is well known that under normal operating conditions, the biomass concentration remains active and the sludge stability is preserved (i.e. $X > 0$ for all $t \geq 0$) which physically means that part of the polluting agents entering the digester are consumed by the bacterial culture (i.e. $S_{in} - S > 0$) (Alcaraz González *et al.*, 2000). Hence, we can assume for practical operation and control design purposes that $S_{in} - S$ is always positive definite.

IV CONTROLLER DESIGN

We now proceed with the design of the particular controllers (11) for the anaerobic digestion (14). We begin the design calculation of the linear matrices around the nominal values $\left(\widehat{\mu}_{\max}, \widehat{K}_s, \widehat{\alpha}\right)$; thus, it is straightforward to show that

$$\begin{aligned} A_0 &= \widehat{\mu}(w_1) \begin{pmatrix} -\frac{1}{\alpha}(\Theta+1) & -\frac{1}{Y} \\ \frac{1}{\alpha}\Theta & 0 \end{pmatrix}, \quad (17) \\ C_0 &= \begin{pmatrix} 1 & 0 \end{pmatrix}, \\ B_0 &= (w_2 - w_1) \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \\ \Theta &= (w_2 - w_1) \frac{\widehat{K}_s}{\widehat{\mu}_{\max}} \frac{\widehat{\mu}(w_1)}{w_1^2}, \end{aligned}$$

while the derivation of the observable immersions depends upon the particular exosystems. These are developed in the following subsections.

A Regulation control

In this section we consider the regulation of the COD concentration, S, around a constant predetermined set-point, S_r, by manipulating the dilution rate, D, with a constant disturbance, S_{in}. It is straightforward to show that both reference and disturbance can be described by a linear exosystem ($\dot{w}_1 = 0$, $\dot{w}_2 = 0$, $w_1 = S_r$ and $w_2 = S_{in}$). In this case, the FIB Eqs. (4) are

$$\begin{aligned} \dot{\pi}_1(w,\lambda) &= (\mu(\pi_2(w)) - \alpha\gamma(w,\lambda))\,\pi_1(w,\lambda) \\ 0 &= (w_2 - \pi_2(w))\,\gamma(w,\lambda) \\ &\quad - \frac{\mu(\pi_2(w))\,\pi_1(w,\lambda)}{Y} \\ 0 &= \pi_2(w) - w_1 \end{aligned}$$

whose solution is given by

$$\begin{aligned} \pi_1(w,\lambda) &= \frac{Y(w_2 - w_1)\,\pi_{10}}{\alpha\pi_{10} + [Y(w_2 - w_1) - \alpha\pi_{10}]\,e^{-\mu(w_1)t}} \\ \gamma(w,\lambda) &= \frac{1}{Y} \frac{\mu(w_1)\,\pi_1(w,\lambda)}{w_2 - w_1} \end{aligned}$$

where $\pi_1(w,\lambda) := X_{ss}$, $\pi_2(w_1) := S_r = w_1$, $\gamma(w,\lambda) := D_{ss}$ and π_{10} is the initial condition for X when $S = w_1$. One can find an exact immersion free of uncertain parameters for $\gamma(w,\lambda)$ similar to (6) with $\zeta \in \mathcal{R}^3$ and $\varphi(\zeta) = col(\zeta_1\zeta_2, \zeta_2\zeta_3, \zeta_2\zeta_3)$; however, it is clear that for sufficiently large times, both mappings, X_{ss} and D_{ss}, reach a steady state (i.e. $\pi_1(w,\lambda) = Y(w_2 - w_1)/\alpha$ and $\gamma(w,\lambda) = \mu(w_1)/\alpha$) and as a consequence, the resulting immersion is a linear system such as (13) of dimension one with $\Phi = 0$ and $H = 1$; i.e.

$$\dot{\zeta}_1 = 0 \quad , \quad \gamma(w,\lambda) = \zeta_1. \tag{18}$$

It can be easily verified that the necessary error feedback controller is a linear controller of dimension three to regulate the COD concentration by manipulating D. The controller's practical implementation requires the measurement of S only and the calculation of the immersion and the nominal matrices A_0, B_0 and C_0.

B Oscillatory disturbance rejection

Many processes experience periodic disturbances due to natural cycle times of upstream processes or other cyclical environmental influences such as diurnal temperature fluctuations. In wastewater treatment plants, for example, the feed flow composition can exhibit strong diurnal variations (Buttler *et al.*, 1995). For this reason, in the AD process (14), we consider the regulation of the COD concentration, S, around a predetermined constant set-point, S_r, under the influence of persistent periodic disturbances of the inlet COD concentration which is represented by $S_{in} = \overline{S}_{in} + A_{S_{in}} \sin\left(\frac{2\pi}{T}t + \theta\right)$, where T is the time period and $\overline{S}_{in}$, $A_{S_{in}}$ and θ are unknown parameters. It is straightforward to show that these disturbances can be described by a linear exosystem $\dot{w} = s(w)$, $S_r = w_1$, $S_{in} = w_2$, where $s(w) = \frac{2\pi}{T}\begin{pmatrix} 0 & w_3 & w_4 & -w_3 \end{pmatrix}^T$. In this case, the error is $e = S - S_r = S - w_1$ which is obviously zero when $S = w_1 =: \pi_1(w)$. Then, by using Eq. (4), we obtain both, the steady state input and the steady state biomass:

$$\pi_1(w,\lambda) = \frac{\pi_{10}}{e^{-\mu(w_1)t} + \frac{\alpha\mu(w_1)\pi_{10}}{Y}\int_0^t \frac{e^{-\mu(w_1)(t-\tau)}}{w_2 - w_1}d\tau}$$

$$\gamma(w,\lambda) = \frac{1}{Y}\frac{\mu(w_1)\pi_1(w,\lambda)}{w_2 - w_1}$$

whereas one can find an exact immersion free of uncertain parameters given by

$$\dot{\zeta} = \varphi(\zeta) \quad , \quad \gamma(w,\lambda) = \zeta_1. \tag{19}$$

that can be used to devise a dimension seven nonlinear controller to keep the COD concentration around a given set point under the influence of persistent periodic disturbances. Here,

$$\varphi(\zeta) = col\left(\zeta_1\zeta_2, \left(-\zeta_2\zeta_3 - \frac{2\pi}{T}\zeta_5 + \zeta_4^2\right), \zeta_2\zeta_3, \left(\frac{2\pi}{T}\zeta_5 - \zeta_4^2\right), \left(-\frac{2\pi}{T}\zeta_4 - \zeta_4\zeta_5\right)\right).$$

C Tracking control

Let us consider a constant disturbance and a given reference which can be described by a linear dynamic system called exosystem of the form $\dot{w} = Sw$, where $w \in \mathcal{R}^{r+1}$, $S_r = w_r$ and $S_{in} = w_{r+1}$. The dynamic matrix $S \in \mathcal{R}^{(r+1)\times(r+1)}$, has the special form

$$S = \begin{pmatrix} \phi & 0 \\ 0 & 0 \end{pmatrix}, \tag{20}$$

where $\phi \in \mathcal{R}^{r\times r}$ may be any matrix such that the pair $\left[\phi, \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}_{1\times r}\right]$ is observable. In this particular case, the solution of the FIB equations yields

$$\pi_1(w,\lambda) = \frac{\pi_{10}e^{\int_0^t \mu(w_1)d\lambda}}{1 + \frac{\alpha\pi_{10}}{Y}\int_0^t \frac{(w_{r+1}-w_1)^{\alpha-1}\mu(w_1)e^{\int_0^\tau \mu(w_1)d\sigma}}{(w_{r+1,0}-w_{1,0})^\alpha}d\tau}$$

$$\gamma(w,\lambda) = \frac{\dot{w}_1 + \frac{1}{Y}\mu(w_1)\pi_1(w,\lambda)}{w_{r+1} - w_1} \tag{21}$$

It is not to big a problem to find an exact immersion free of uncertain parameters for this particular study case but the result is a high dimensional immersion which may demand significant computation effort and time in the controller design. Instead, one can find a simpler immersion by looking at equation (21) where it is clear that as $t \to \infty$, $\pi_1(w,\lambda)$ attains a "pseudo" steady-state which depends exclusively on the exosystem dynamic behavior. Moreover, one can easily show that for slow time varying references, $\gamma(w,\lambda)$ can be satisfactorily approximated by a linear immersion, which in this case of tracking an oscillatory reference with period T, $S_r = \overline{S}_r + A_{S_r}\sin\left(\frac{2\pi}{T}t + \phi\right)$, results in a dimension five immersion given by $\dot{\zeta} = \Phi\zeta$, $\gamma(w,\lambda) = H\zeta$, where

$$\Phi = \frac{2\pi}{T}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad , \quad H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}. \tag{22}$$

V RESULTS AND DISCUSSIONS

Several closed-loop simulation runs were performed to asses the performance and robustness of the robust error feedback controller developed for the three study cases. The nominal matrices A_0, B_0 and C_0 were calculated by using the parameter values reported in (Alcaraz González *et al.*, 2000) (for the sake of completeness, these are listed in Table 1). In all study cases, the proposed controllers had the form of equation (11), where the feedback gain, $K = \begin{pmatrix} 0.40 & 0.63 \end{pmatrix}$, was calculated such that $(A_0 + B_0K)$ is Hurwitz, with eigenvalues $(-8, -5.6)$, while the observer gains, G_1 and G_2, were calculated using LQG techniques such that

Table 1: Parameters nominal values and variation through simulation time

Parameter	Nominal Value	% of variation from nominal values (Interval of time)							
	$0 \leq t \leq 20$	[20,30)	[30,40)	[40,50)	[50,60)	[60,70)	[70,80)	[80,90)	[90,100)
$\mu_{\max}$, d^{-1}	1.25	−15%	–	–	–	–	–	–	–
K_s, $kg \cdot m^{-3}$	4.95	0%	20%	–	–	–	–	–	–
α, dimensionless	0.5	0%	–	20%	–	–	–	–	–
$1/Y$ kg/kg	6.6	0%	–	–	-25%	–	–	–	–
S_r or $\overline{S}_r$, $kg \cdot m^{-3}$	2.5	0%	–	–	–	-40%	–	–	40%
S_{in} or $\overline{S}_{in}$, $kg \cdot m^{-3}$	10	0%	–	–	–	–	30%	–	–
$A_{S_{in}}$ *, $kg \cdot m^{-3}$	1	0%	–	–	–	–	–	20%	–
A_{S_r} **, $kg \cdot m^{-3}$	0.5	0%	–	–	–	–	–	-50%	–

* Only for the oscillatory disturbance rejection case

** Only for the oscillatory tracking control case

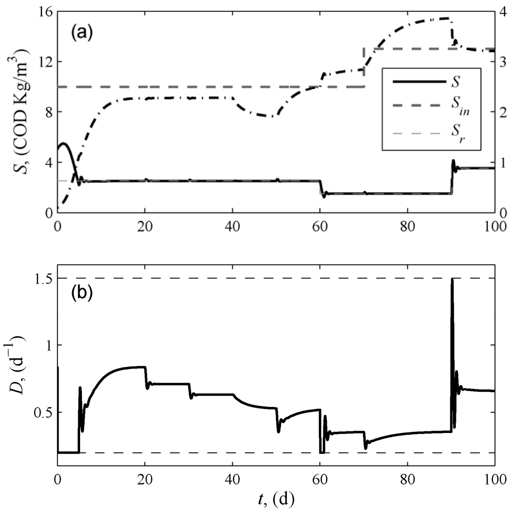

Figure 1: Regulation control:(a) Sustrate and biomass concentrations. (b) Dilution rate.

matrix (12) was Hurwitz, depending on the particular immersion.

In order to test the robustness properties of the proposed controllers, high substrate concentration and low biomass were imposed at the start-up of the simulation runs, whereas parametric variations were induced during simulations as reported in Table 1. These variations were relatively large (from 15 to 50% of the nominal values) and describe actual operating conditions in real AD plants. For instance, variations in kinetic parameters ($\mu_{\max}, K_s, Y$) may describe biological problems due to poisoning or biological stress conditions. On the other hand, variations in α may describe hydrodynamic malfunctions or cells death, while variations in S_{in} describes the usual fluctuations and changes in influent concentrations. Finally, step changes in substrate reference concentration were also induced during the simulation experiments.

Regulation control: Figure 1 shows the closed-loop response of the proposed controller using immersion (18), with observer matrices $G_1 = \begin{pmatrix} 2.50 & -6.98 \end{pmatrix}^T$ and $G_2 = -1.73$. As seen, the proposed controller was able to rapidly drive the COD to its set-point in the face of all the parameter variations (the output error was kept around zero or identically equal to zero most of simulation run). One can also see that the input variable, D, saturated at different times without serious consequences on the controller performance. It is worth noticing that as a result of immersion (18), controller (11) had a proportional-integral (PI)-like form with an error filter, where the immersion state, ζ, described the integral action, while ξ described the error filter.

Oscillatory disturbance rejection: In this case, we applied controller (11) and used the nonlinear immersion (19) with observer matrices $G_1 = \begin{pmatrix} 3.57 & -8.99 \end{pmatrix}^T$ and $G_2 = \begin{pmatrix} -3.13 & -3.60 & -2.71 & -0.10 & 0.46 \end{pmatrix}^T$, with an oscillation period of $4d$. Clearly, the proposed controller handled quite well the persistent periodic disturbance and attenuated the effect of varying parameters to yield an oscillatory output profile around the desired set-point with a rather small magnitude. One can also see in Fig. 2 that the controller generated oscillatory control actions that respected the input constraints with rare excursions to saturation.

Tracking control: Finally, Fig. 3 shows the closed-loop response of the output and manipulated input under the robust nonlinear controller for the oscillatory tracking problem with oscillation period $T = 2d$, where a linear immersion (22) was used to devise a controller (11) with observer gains $G_1 = \begin{pmatrix} 5.18 & -11.25 \end{pmatrix}^T$ and $G_2 = \begin{pmatrix} -5.77 & -2.21 & 2.03 \end{pmatrix}^T$. As can be seen, the oscillatory reference was perfectly tracked by the proposed robust controller. As expected, the controller generated oscillatory control actions but these respected the imposed constraints for most of the simulation run.

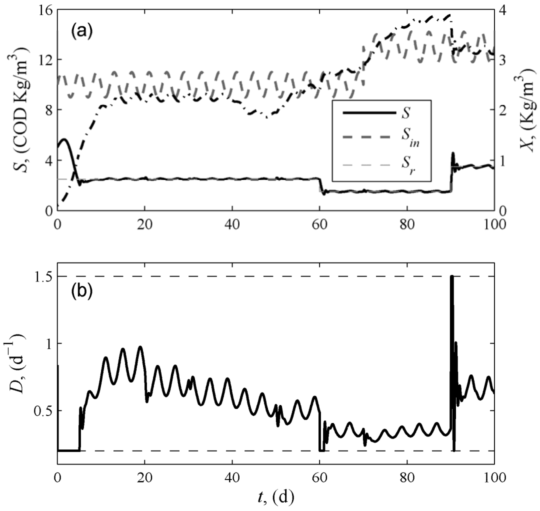

Figure 2: Oscillatory disturbance rejection: (a) Sustrate and biomass concentrations. (b) Dilution rate.

VI CONCLUSIONS

A robust control scheme was proposed as a potential solution to nonlinear bioprocess regulation . It is composed of an error feedback controller and a nonlinear estimator which allows us to track constant and/or oscillatory profiles while attenuating the disturbances and maintaining the stability conditions. The performance of the robust regulator has been examined through numerical simulations under various uncertainties and external disturbances. The proposed structure has shown to maintain good stability and robustness properties in the face of set-point changes, parameter uncertainty and load disturbances.

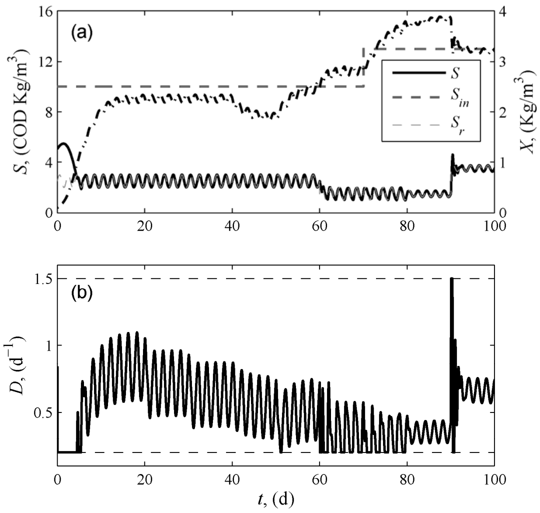

Figure 3: Tracking control: (a) Sustrate and biomass concentrations. (b) Dilution rate.

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Received: July 12, 2009.
Accepted: November 11, 2009.
Recommended by Subject Editor Jorge Solsona.