NEHARI PROJECTION AND SOS IMPLEMENTATION IN REAL–TIME STABLE IDENTIFICATION

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***Abstract*— An adaptive identification algorithm based on Second Order Section (SOS) model structures is presented. The procedure guarantees stable transfer functions whenever the actual physical plant is stable, due to an optimal Nehari approximation step performed analytically online. The procedure is suitable for real time applications. Some synthetic and experimental examples illustrate the proposed algorithm.**

***Keywords*— Nehari Projection, SOS Implementation, Real Time Identification**

I. INTRODUCTION

Adaptive identification algorithms have been used in the area of adaptive control systems for a very long time, both for feedback (FB) and/or feedforward (FF) approaches (Goodwin and Sin, 1984; Tao, 2003). Usually for simplicity and computational speed in real time applications, parametric linear schemes have been implemented: RLS, NLMS, FXLMS, FULMS, as in the case of Active noise control (Kuo and Morgan, 1995), for example. Nevertheless the traditional assumptions in adaptive control: lack of perturbations or high frequency uncertain dynamics and minimum phase models, have generated at the end of the 80's an intense work in the area of robustness of adaptive laws (Tao, 2003; Narenda, 1986; Ioannou and Sun, 1996). These have been extensively studied since then, and an excellent survey in this area can be found in Ortega and Tang (1989).

Still then in adaptive identification, the stability of the resulting IIR model is generally not guaranteed, causing serious practical problems particularly in FF implementations. There are methods to convert IIR to FIR like the *Nehari shuffle* (Kootsookos *et al.*, 1992) and a recent LMI optimal version in Yamamoto *et al.* (2002), but the error is usually greater and requires a larger number of parameters in general. The use of IIR filters instead has the potential to decrease the identification error due to the fact that it includes the pole dynamics. In addition, this class of filters are in certain applications more efficient in modelling signals and require smaller model orders (Rao, 1993). Therefore an IIR filter that can guarantee a stable behavior and can be used in real time applications is a necessary tool in practical situations.

On the other hand, numerical problems also arise in real time applications, depending on the structural representation of the model. Take for example an 11th. order stable filter implemented with three different model structures: zero–pole (ZP), state space (SS) and transfer function (TF), the latter in terms of numerator and denominator coefficients, as follows:

$$\text{(ZP)} \quad \prod_{i=1}^{m} \frac{(z_i z^{-1}-1)}{(p_i z^{-1}-1)}, \qquad \text{(TF)} \quad \frac{\sum_{i=0}^{m} z^{-i} b_i}{\sum_{i=0}^{m} z^{-i} a_i}$$

$$\text{(SS)} \quad \begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}$$

The complexity of each model is $\mathcal{O}(m^2)$ in the case of SS and $\mathcal{O}(m)$ in the other two cases, therefore from this point of view, the ZP and TF structures are more efficient. Nevertheless, it is a well known fact that the pole locations in the case of the TF structure, particularly in high order models, are significantly modified, even producing unstable poles ($|p_i| > 1$), as illustrated in Table 1. On the other hand, it is easier to use the TF representation as the difference equation which implements the filter in real time, as follows:

$$y_k = \frac{1}{a_0} \quad [b_0\, u_k + \cdots + b_m\, u_{k-m} \\ -a_1\, y_{k-1} - \cdots - a_m\, y_{k-m}] \qquad (1)$$

Therefore, the TF representation has advantages in terms of complexity and implementation, but serious disadvantages in terms of perturbations of pole locations, at least in high order models.

The solution to this problem is obtained by a series connection of Second Order Sections (SOS), which is an adequate way of implementing filters in real time. The SOS structure is numerically more efficient than the plain TF structure due to the fact that it has a 2nd. order numerator and denominator, therefore preserving the original pole–zero locations. In addition, cascade-forms of SOS provide an attractive realization

Table 1: Absolute value of poles of a discrete–time system represented in zero–poles (ZP), state–space (SS) and transfer function (TF).

ZP	SS	TF
0.0034	0.0034	0.0034
0.9975	0.9975	1.0295
0.9975	0.9975	1.0295
0.9949	0.9949	1.0128
0.9949	0.9949	1.0128
0.9607	0.9607	0.9924
0.9608	0.9608	0.9924
0.9802	0.9802	0.9646
0.9995	0.9995	0.9646
0.9995	0.9995	0.9434
0.9961	0.9961	0.9434

for adaptive IIR filters because the stability of the filter parametrization is easily monitored, and because filter pole locations are readily obtained from the adapted parameters with low computational cost (Williamson *et al.*, 1995).

In the previous example, the SOS' pole locations coincides with the ZP and SS structures. Furthermore it is still $\mathcal{O}(m)$ and each SOS can be implemented as a difference equation connected in series with all other SOS', as follows:

$$\frac{Y(z)}{U(z)} = \prod_{i=1}^{m/2} \frac{z^{-2}b_2^i + z^{-1}b_1^i + b_0^i}{z^{-2}a_2^i + z^{-1}a_1^i + 1} \qquad (2)$$

and each SOS is implemented as a 2nd. order difference equation:

$$y_k^i = b_0^i\ u_k^i + b_1^i\ u_{k-1}^i + b_2^i\ u_{k-2}^i - a_1^i\ y_{k-1}^i - a_2^i\ y_{k-2}^i$$

where $a_0^i = 1$ for simplicity.

There are many applications where a stable adaptive real time identification is needed, one of them being Active Noise Control (ANC) (Lueg, 1934; Nelson and Elliot, 1992; Kuo and Morgan, 1995). There, significant noise attenuation can be achieved through FB and/or FF controllers. In the first case, there are many well known limitations of the feedback loop that produces a poor performance. These performance limitations are mainly due to the non-minimum phase nature of the plant (see Freudenberg and Looze, 1985; Seron *et al.*, 1997 and also Hong and Bernstein, 1998) and its revision in Freudenberg *et al.* (2003), which in turn is derived from the time delay of sound propagation, e.g. acoustic tubes. Instead, a FF filter performs better because it is not restricted to the loop limitations. In this kind of application, the FF controller acts as a real time identifier of the acoustic noise signal received by the error microphone at the end of the tube, in order to cancel it at that point. Usually an adaptive identification scheme is used, which can produce an unstable behavior in many situations. More details will be given in the application example at the end of this work. A complete experimental study of a hybrid – FF/FB controller applied to ANC in a tube can be found in Cugueró *et al.* (2007).

As a consequence, a convergent adaptive identifier with guaranteed stable behavior and numerical robustness is very useful in these situations. Such an algorithm will be described in this work. Numerical robustness is achieved by the use of SOS' and the stability of each section is guaranteed by a stable Nehari projection (Ball *et al.*, 1990), which provides the nearest (optimal) stable model to a possibly unstable one. Due to the fact that the objective of this procedure is to implement it in real time situations, the Nehari projection is developed in analytical form.

The paper is organized as follows. Next section presents the background material: an analytic version of the Nehari stable projection and the adaptive identification procedure. Section III. presents the main result of this work, followed by Section IV. which illustrates different aspects and limitations of this procedure through several examples. Final conclusions are drawn in Section V.

II. PRELIMINARY RESULTS

A. Nehari's stable projection

Nehari's result is well known in the area of systems and control (Ball *et al.*, 1990). It produces, both for discrete and continuous time systems, the optimal stable projection of an unstable system[1]. Furthermore, the optimality implies that the resulting error is an all–pass filter with a gain corresponding to the highest Hankel singular value of the original system. It can be stated as follows:

Theorem II.1 *Given a completely unstable system* $U(x)$*, its optimal stable projection has the following solution:*

$$\inf_{S\in\mathcal{H}_\infty} \|U(x) - S(x)\|_\infty = \|E(x)\|_\infty = \bar{\sigma}$$

where $\mathcal{H}_\infty$ *is the Hardy space corresponding to causal transfer functions,* $E(x)$ *is an all–pass filter,* $\bar{\sigma}$ *is the largest Hankel singular value of* $U^*(x) \in \mathcal{H}_\infty$ *and variable* x *holds for either* s *or* z*, both complex variables corresponding to the Laplace or Z–transforms of continuous or discrete time systems, respectively.*

Here both, the stable projection and the corresponding error in the case of 2nd. and first order models have been computed analytically. Due to the fact that the purpose is to achieve a real time fast implementation, without loss of generality, a discrete time model has been considered. Therefore, given a SOS with poles

[1]Actually it produces the optimal stable projection of a totally unstable model, i.e. with all its poles unstable. In the general case, the projection applies only to the unstable poles, and the stable ones remain the same.

p_1 and p_2:

$$\begin{aligned} f(z) &= \frac{z^{-2}b_2 + z^{-1}b_1 + b_0}{z^{-2}a_2 + z^{-1}a_1 + 1} \\ &= \frac{\cdots}{(1+p_1\ z^{-1})(1+p_2\ z^{-1})} \qquad (3) \\ p_1 &= \frac{a_1 - \sqrt{a_1^2 - 4a_2}}{2} \\ p_2 &= \frac{a_1 + \sqrt{a_1^2 - 4a_2}}{2} \end{aligned}$$

it's stability condition needs to be verified, and there are 3 cases:

Stable poles: No modification is performed, i.e. $f_s(z) = f(z)$.

Stable–Unstable poles: The SOS is separated into it's stable and unstable parts, as follows:

$$f(z) = \underbrace{\frac{x_0 + x_1\ z^{-1}}{1 + p_1\ z^{-1}}}_{H_u} + \underbrace{\frac{1 + x_2\ z^{-1}}{1 + p_2\ z^{-1}}}_{H_s}$$

$$\left\{ \begin{array}{l} x_0 = b_0 - 1 \\ x_1 = \frac{p_1^2 + x_o a_2 + b_2 - p_1 b_1}{\sqrt{a_1^2 - 4a_2}} \\ x_2 = \frac{p_2 b_1 - a_2 - x_o p_2^2 - b_2}{\sqrt{a_1^2 - 4a_2}} \end{array} \right.$$

where the unstable pole has been assumed as $|p_1| > 1$. Similar results are derived by simply changing $p_1 \leftrightarrow p_2$ in the previous equations, if $|p_2| > 1$.

The optimal Nehari stable projection of the unstable term is X_{opt} with a constant approximation error over all frequencies $\bar{\sigma}$, producing a stable optimal approximation of the SOS $f_s(z)$:

$$\begin{aligned} X_{opt} &= \frac{x_0 - x_1 p_1}{1 - p_1^2} \\ \bar{\sigma} &= x_1 - p_1 X_{opt} \\ f_s(z) &= X_{opt} + H_s(z) \\ &= \frac{(X_{opt}+1) + (x_2 + p_2 X_{opt})z^{-1}}{1 + p_2 z^{-1}} \end{aligned}$$

Unstable poles: Here both poles are unstable, i.e. $|p_1| > 1$ and $|p_2| > 1$. The optimal Nehari stable projection of $f(z)$ is:

$$f_s(z) = \frac{n_0 + n_1 z^{-1}}{1 + d_1 z^{-1}}$$

$$\left\{ \begin{array}{l} n_0 = b_0 - a_2 d_1 \bar{\sigma} \\ n_1 = b_1 - a_1 b_0 - a_2 \bar{\sigma} - d_1 \left[\bar{\sigma} a_1 (1 - a_2) - b_0\right] \\ d_1 = \frac{a_2 b_1 - a_1 a_2 b_0 + \bar{\sigma}(1 - a_2^2)}{a_2[\bar{\sigma} a_1 (1 - a_2) - b_0] + b_2} \end{array} \right.$$

and the constant approximation error over all frequencies is the solution to the following quadratic equation:

$$\alpha \bar{\sigma}^2 + \beta \bar{\sigma} + \gamma = 0$$

$$\left\{ \begin{array}{l} \alpha = 1 - a_1^2 (a_2 - 1)^2 + a_2^2 (a_2^2 - 2) \\ \beta = a_1 (a_2 - 1)^2 (b_0 + b_2) - b_1 + a_2 b_1 (1 + a_2 - a_2^2) \\ \gamma = a_1 a_2 b_0 b_1 - (a_2 b_0)^2 - a_2 b_1^2 - b_0 b_2 a_1^2 \\ \qquad +2 a_2 b_0 b_2 + a_1 b_1 b_2 - b_2^2 \end{array} \right.$$

The choice of $\bar{\sigma}$ corresponds to the stable case, i.e. $|d_1| < 1$.

B. Adaptive identification procedure

Without loss of generality, any robust adaptive identification procedure could be used, due to the fact that the algorithm relies on the SOS structure and the optimal Nehari stable approximation (in closed form). Here, it has been decided to test the algorithm using the well known Switching σ-modification (Tao, 2003). This adaptive identification algorithm, can be stated as follows (Ioannou and Sun, 1996):

Given the i–th SOS as in Eq. (3):

$$f_i(z) = \frac{z^{-2}b_2^i + z^{-1}b_1^i + b_0^i}{z^{-2}a_2^i + z^{-1}a_1^i + 1}$$

define the parameters and regressors as follows:

$$\theta_k^i = \begin{bmatrix} b_0^i \\ b_1^i \\ b_2^i \\ a_1^i \\ a_2^i \end{bmatrix}, \quad r_k^i = \begin{bmatrix} u_k^i \\ u_{k-1}^i \\ u_{k-2}^i \\ -y_{k-1}^i \\ -y_{k-2}^i \end{bmatrix}$$

Assume that a bound M^i of the norm of the true parameter Θ^i is known, *i.e.* $\|\Theta^i\| \leq M^i$, where $\|\cdot\|$ is the Euclidean norm of a vector. Then the algorithm which produces the i–th SOS model y_{mk}^i at time $t = t_k$, and the parameter update is:

$$y_{mk}^i = [r_k^i]^T \theta_{k-1}^i \qquad (4)$$

$$K_k^i = \frac{c\ r_k^i}{\alpha + \left\|r_k^i\right\|^2} \qquad (5)$$

$$\epsilon_k^i = K_k^i \left[y_k^i - y_{mk}^i\right], \qquad (6)$$

$$\theta_k^i = (1 - \sigma_k^i)\theta_{k-1}^i + \epsilon_k^i, \qquad (7)$$

where (c, α) are the convergence gain and *forgetting* factor parameters, both chosen by the designer with $0 < c < 1$ and

$$\sigma_k^i = \left\{ \begin{array}{ll} \sigma_0 & \text{if} \quad \|\theta_k^i\| \geq 2M^i \\ 0 & \text{if} \quad \|\theta_k^i\| < 2M^i, \end{array} \right.$$

and where $0 < \sigma_0 < (1 - c)/2$. Next, and due to the fact that this algorithm cannot assure the stability of the SOS, the Nehari projection algorithm described above is applied in order to obtain the estimates θ_k^i of the i–th SOS.

III. MAIN ALGORITHM

Next, the identification algorithm is presented based on a system modelled as a chain of N SOS. The input to the first one is the **physical** input of the system and the output of the N-th SOS its **physical** output.

A. I/O propagation

The algorithm identifies each SOS separately connecting the results altogether. To this end, the inputs of

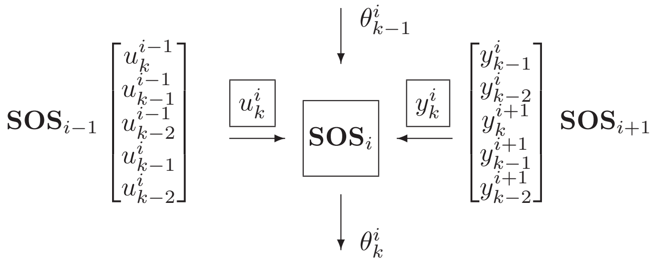

Figure 1: Inputs and outputs propagation scheme.

the i–th SOS need to be defined as the outputs propagated from the $(i-1)$–th SOS. Similarly, the outputs from the i–th SOS are (inversely) propagated from the $(i+1)$–th SOS, as follows:

$$u_k^i = \left[u_k^{i-1} \quad u_{k-1}^{i-1} \quad u_{k-2}^{i-1} \quad -u_{k-1}^i \quad -u_{k-2}^i\right] \theta_{k-1}^{i-1}$$

$$y_k^i = \frac{1}{b_0^{i+1}} \left\{ \left[0 \quad -y_{k-1}^i \quad -y_{k-2}^i \quad y_{k-1}^{i+1} \quad y_{k-2}^{i+1}\right] \theta_{k-1}^{i+1} + y_k^{i+1} \right\}$$

Here, the first input coincides with the physical input $u_k^1 = u_k$ and the same for the last (N–th) output $y_k^N = y_k$. In the propagation of the input signal u_k^i, the updated value of the parameters θ_k^{i-1} can also be used. The propagation scheme has been represented in Fig. 1 and is part of the main algorithm in Fig. 2 as the blocks `Propaga_Y` and `Propaga_U`. Referring to the initialization, the values $u_k^i = y_k^i = 0$, $\forall i$, $\forall k < 0$ could be adopted, and an initial off-line identification could provide the initial values for the estimated parameters.

B. Algorithm

The complete flow diagram is illustrated in Fig. 2. The external loop goes from time t_k to t_M and produces the model output y_{mk} at time t_k. The propagation scheme has been explained previously and produces the inputs and outputs for each SOS, here denoted generically as the i–th SOS. These will be part of the regressor vector r_k^i used in the next step. The identification (`ID SOS`) has been explained in Section II. B. and produces the i–th SOS updated parameter vector θ_k^i at time t_k. As explained previously, at this stage any convergent identification algorithm can be applied. Next, and depending on the stability of the SOS due to the values of the parameters θ_k^i, the projection procedure, explained in Section II. A., is applied at stage `Project SOS` in Fig. 2. It produces a stable set of parameters $\theta_{k-stable}^i$ that will be used to produce the model output y_{mk}^i, according to Eq. (4). An error term $y_k^i - y_{mk}^i$ is also generated to keep track of the convergence of the whole procedure. The number of SOS sections is arbitrary, and the total order of the system's model can be odd. In that case, the remaining first order section can be easily projected in case it goes unstable at some point in the identification procedure. This has been explained in Section II. A. as a particular case when only one of the poles of a SOS section is unstable.

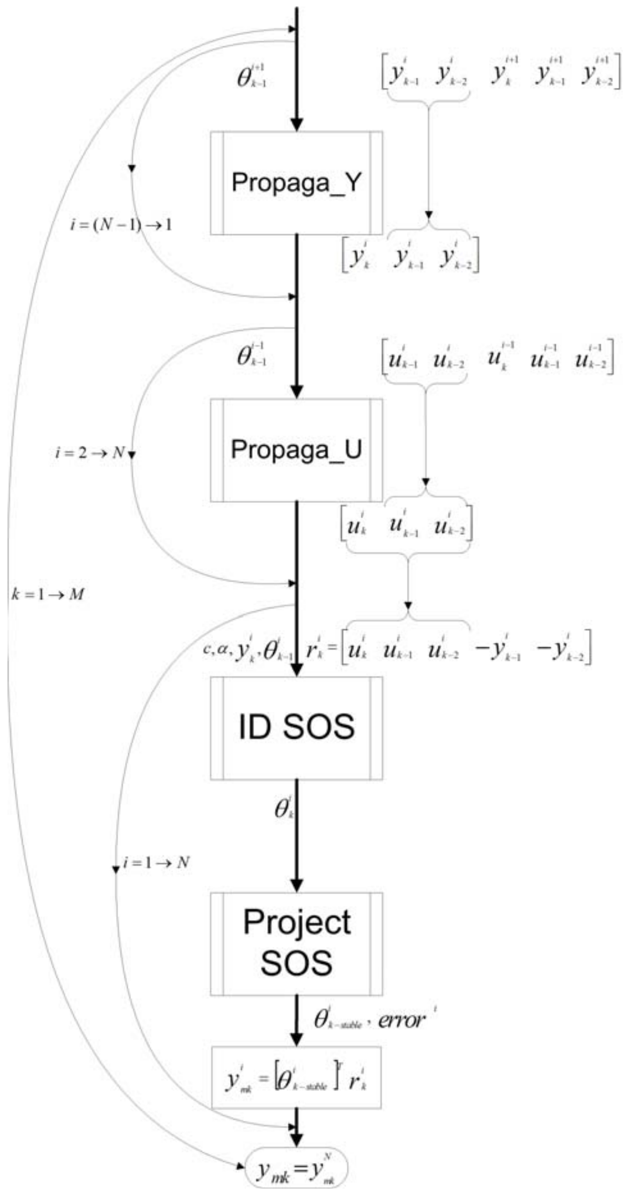

Figure 2: Algorithm flow diagram.

C. Analysis

The effects of both, the propagation scheme (`Propaga_Y`, `Propaga_U`) and the Nehari projection (`Project SOS`), on each SOS can be represented by a modelling error process, and the system can be described by the following difference equation:

$$y_k^i = b_0^i\, u_k^i + b_1^i\, u_{k-1}^i + b_2^i\, u_{k-2}^i - a_1^i\, y_{k-1}^i - a_2^i\, y_{k-2}^i + \Delta_k^i$$

where Δ_k^i represents the modelling and measurement errors, and verifies

$$\left|\Delta_k^i\right| \leq \delta_k^i \left\|r_k^i\right\|^2 + \mu_k^i, \quad \delta_k^i \in \mathcal{L}_\infty, \quad \mu_k^i \in \mathcal{L}_\infty.$$

Here δ_k^i takes into account the parameter errors in section i and μ_k^i the effect of the propagation errors due to other sections. As a consequence of this inequality and according to Tao (2003), θ_k^i is bounded and $\left\{\left[y_k^i - y_{mk}^i\right]^2 / (\alpha + \left\|r_k^i\right\|^2)\right\} \in \mathcal{L}_2$.

Remark III.1 *The introduction of the term* $-\sigma_k^i \theta_{k-1}^i$ *in the adaptive identification algorithm (a modification in fact of the well known gradient algorithm) ensures the boundedness of* θ_k^i *and that* $\left\{\left[y_k^i - y_{mk}^i\right]^2 / (\alpha + \left\|r_k^i\right\|^2)\right\} \in \mathcal{L}_2$. *This effect, that is local for each SOS, produces a robust, with respect to the initial estimates of the parameters, identification algorithm for the whole cascade.*

It must be pointed out that if any other robust identification algorithm is used instead of the Switching σ*-modification, similar results will be obtained.*

D. Numerical issues

The online Nehari projection performed analytically takes care of the possible appearance of unstable poles along the real time identification procedure. Furthermore, the SOS model structure provides greater numerical stability to the implementation. Nevertheless, as with any other algorithmic solutions to a particular problem, there are numerical issues to be taken into consideration, both for programming and implementation. These are as follows:

- Non minimum phase (NMP) zeros: These zeros produce unstable behaviors when propagating the output signals y_k^i to produce the equivalent i–th SOS inputs u_k^i (see Fig. 1).
- Model order reduction: The Nehari stable approximation reduces the order of the i–th SOS unstable model according to the number of unstable poles it has, i.e. for 1 unstable pole, to order 1; for both unstable poles, to a constant (see Section II. A.).

These are solved as follows:

- The input and output signals are scaled at each time step k in order not to exceed certain absolute value limits.
- The procedure here is to perform the stable projection (and hence the SOS order reduction) at step k, but at next step, return to the previous data that generated a stable SOS in the next previous step. The argument here comes from the fact that the actual system is always open loop stable.

IV. EXAMPLES

A. Example 1.

This example illustrates the performance of the proposed algorithm with respect to the initial errors in

Table 2: Coefficients of the SOS represented as transfer functions.

	SOS1	SOS2	SOS3
b_0	1.4286	1	1
b_1	2.4426	0.39854	-1.3084
b_2	1.2422	1.0125	0.79505
a_0	1	1	1
a_1	1.2682	0.41029	-0.82137
a_2	0.48484	0.50966	0.57812

Table 3: Initial coefficients of the identification algorithm represented as transfer functions.

	SOS1	SOS2	SOS3
b_0	1.3857	1.1167	1.0917
b_1	2.0916	0.4562	-1.2241
b_2	1.0648	0.8643	0.8377
a_0	1	1	1
a_1	1.2374	0.3667	-1.0155
a_2	0.4394	0.4455	0.5446

the values of the estimated system parameters θ_0^i, a practical constraint which should always be taken into account. In this case a 6–th. order stable model composed of three SOS has been identified. The coefficients of each of the SOS are given in Table 2. The initial values of the model parameters adopted were randomly generated to differ with those above in approximately 15 %. These values are given in Table 3.

The input to the system is a train of pulses and the model and system outputs are illustrated in Fig. 3. Note that the algorithm behaves well, taking into account the highly demanding input and the relatively high uncertainty in the initial setting of the parameters. The following examples will show that the performance diminishes when the uncertainty and/or the number of SOS in the cascade increase.

Although the Switching σ modification algorithm has been used, similar results were obtained using in the simulations other robust identification procedures such as the Projection algorithm (Tao, 2003). This has also been illustrated in the previous example. This fact shows that the structure of the proposed algorithm: *propagation scheme - robust identification - Nehari projection* is central and the robust identification algorithm applied is of secondary importance.

B. Example 2.

This and the next example are taken from a practical application which uses measurements from an acoustic tube in an active noise control experience. The tube is 4 meters long and has a reference and error microphones located at both extremes (see Fig. 4). In this example, the input signal is produced by an industrial fan and has been measured by the reference microphone located next to it. The output signal has been measured by the error microphone at the other end

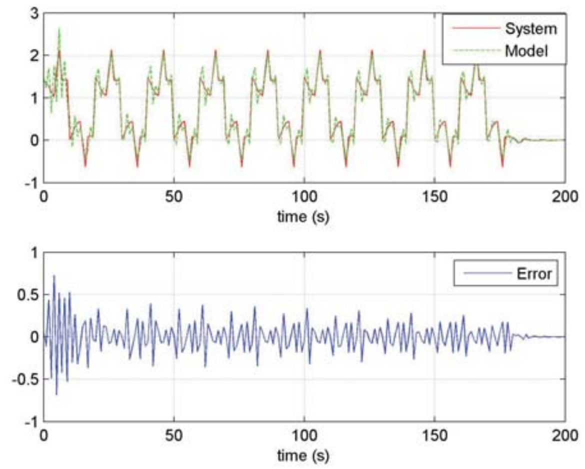

Figure 3: 6–th. order model approximation using the Switching σ modification algorithm.

of the tube, therefore the primary circuit is identified. The identification scheme is based on the Projection algorithm (Tao, 2003) and the initial coefficients of all SOS sections have been computed from an off–line identification of the complete transfer function based on a parametric–nonparametric technique (Parrilo *et al.*, 1999). This is a convenient practical approach so that the algorithm is initiated from a close enough neighborhood of the actual parameters. The type of off-line identification procedure used is irrelevant as long as it produces a sufficiently good model, taking advantage of the fact that it does not need to be implemented in real time. The results are presented in Fig. 5, which evidence a good fit of the experimental data.

C. Example 3.

The next example considers experimental data generated by the same tube, but with the control speaker as the main noise source, which produces a multi–sinusoidal signal. The output is again obtained from the error microphone. The system to be identified is the secondary circuit based on a high order model (40th). Again, a previous off-line identification has been made by means of a parametric–nonparametric robust identification algorithm in Parrilo *et al.* (1999). The online identification scheme is again based on the Projection algorithm and has considered the first 500 data points. The remaining 1000 data points are used as a validation test. Here the main objective is to test the algorithm against numerical errors produced in cases where high order models are used. The fit is good enough and the error can be seen in Fig. 6 to be bounded.

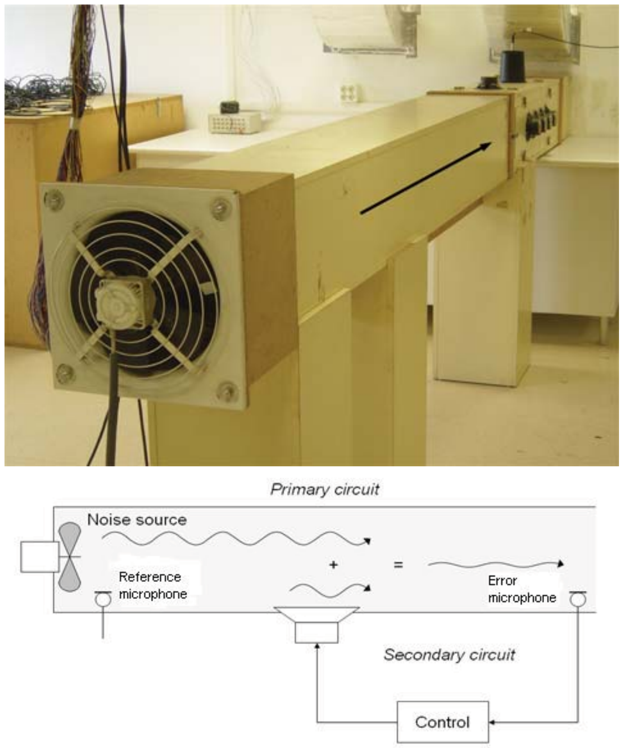

Figure 4: Active noise control experiment and conceptual diagram.

V. CONCLUSIONS

A robust adaptive identification algorithm based on a SOS-cascade realization with forward and backward propagation of the input and output regressors has been presented. Stability of the estimation of each SOS (and as a consequence of the whole cascade) is obtained via a Nehari projection. Simulated and experimental examples illustrate the performance of this algorithm.

VI. ACKNOWLEDGMENTS

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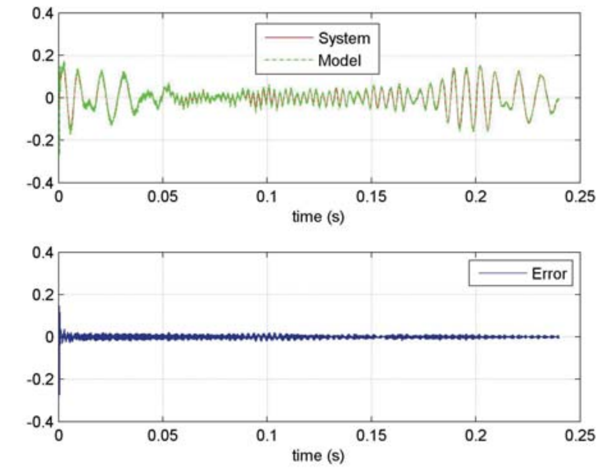

Figure 5: 12th. order experimental data approximation.

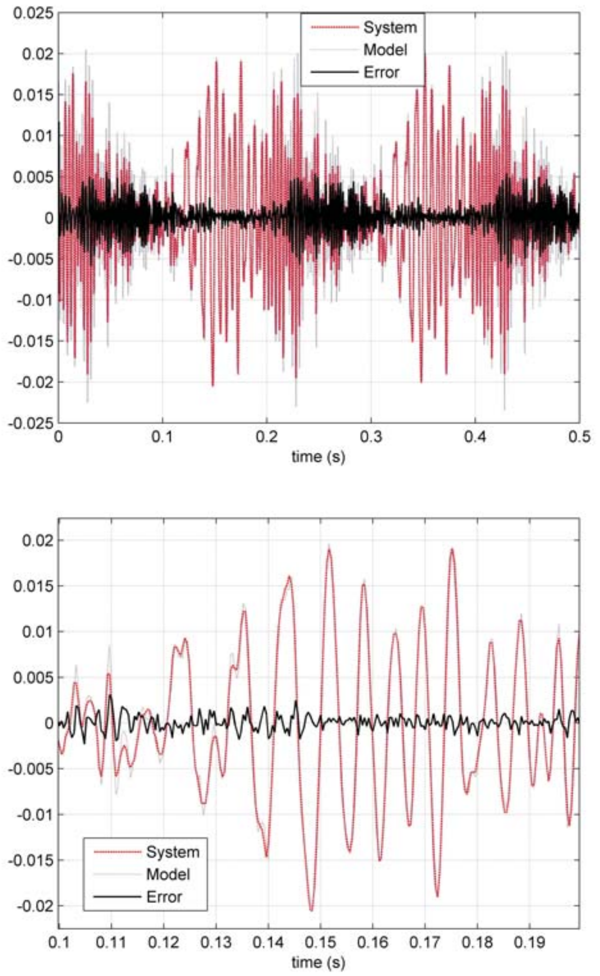

Figure 6: 40th. order experimental data approximation.

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